Sequences and Series

Question 1.
Let Tr be the rth term of an A.P. whose first term is a and the common difference is d. If for some positive integers m, n, $m \neq n$, T m = 1/n and T n = 1/m then (a-d) equals to
(a) 0
(b) 1
(c) 1/mn
(d) 1/m + 1/n

Answer: (a) 0
Given the first term is a and the common difference is d of the AP
Now, Tm = 1/n
$\Rightarrow$ a + (m – 1)d = 1/n ………… 1
and Tn = 1/m
$\Rightarrow$ a + (n – 1)d = 1/m ………. 2
From equation 2 – 1, we get
(m – 1)d – (n – 1)d = 1/n – 1/m
$\Rightarrow$ (m – n)d = (m – n)/mn
$\Rightarrow$ d = 1/mn
From equation 1, we get
a + (m – 1)/mn = 1/n
$\Rightarrow$ a = 1/n – (m – 1)/mn
$\Rightarrow$ a = {m – (m – 1)}/mn
$\Rightarrow$ a = {m – m + 1)}/mn
$\Rightarrow$ a = 1/mn
Now, a – d = 1/mn – 1/mn
$\Rightarrow$ a – d = 0

Question 2.
The first term of a GP is 1. The sum of the third term and fifth term is 90. The common ratio of GP is
(a) 1

(b) 2
(c) 3
(d) 4

Answer: (c) 3
Let first term of the GP is a and common ratio is r.
3rd term = ar^2
5th term = ar^4
Now
$\Rightarrow ar^2 + ar^4 = 90$
$\Rightarrow a(r^2 + r^4) = 90$
$\Rightarrow r^2 + r^4 = 90$
$\Rightarrow r^2 \times (r^2 + 1) = 90$
$\Rightarrow r^2 (r^2 + 1) = 3^2 \times (3^2 + +1)$
$\Rightarrow r = 3$
So the common ratio is 3

Question 3.
If a is the first term and r is the common ratio then the nth term of GP is
(a) $(ar)^{n-1}$
(b) $a \times r^n$
(c) $a \times r^{n-1}$
(d) None of these

Answer: (c) $a \times r^{n-1}$
Given, a is the first term and r is the common ratio.
Now, nth term of GP = $a \times r^{n-1}$

Question 4.
The sum of odd integers from 1 to 2001 is
(a) 10201
(b) 102001
(c) 100201
(d) 1002001

Answer: (d) 1002001
The odd numbers from 1 to 2001 are:
1, 3, 5, ………, 2001
This froms an AP

where first term a = 1
Common difference d = 3 – 1 = 2
last term l = 2001
Let number of terms = n
Now, l = a + (n – 1)d
⇒ 2001 = 1 + (n – 1)2
⇒ 2001 – 1 = (n – 1)2
⇒ 2(n – 1) = 2000
⇒ n – 1 = 2000/2
⇒ n – 1 = 1000
⇒ n = 1001
Now, sum = (n/2) × (a + 1)
= (1001/2) × (1 + 2001)
= (1001/2) × 2002
= 1001 × 1001
= 1002001
So, the sum of odd integers from 1 to 2001 is 1002001

Question 5.
If a, b, c are in AP and x, y, z are in GP then the value of $x^{b-c} \times y^{c-a} \times z^{a-b}$ is
(a) 0
(b) 1
(c) -1
(d) None of these

Answer: (b) 1
Given, a, b, c are in AP
⇒ 2b = a + c ………. 1
and x, y, z are in GP
⇒ $y^2 = xz$ ……….. 2
Now, $x^{b-c} \times y^{c-a} \times z^{a-b} = x^{b-c} \times (\sqrt{xz})^{c-a} \times z^{a-b}$
$= x^{b-c} \times x^{(c-a)/2} \times z^{(c-a)/2} \times z^{a-b}$
$= x^{b-c} + x^{(c-a)/2} \times z^{(c-a)/2+ a -b}$
$= x^{2b+(c+a)} \times z^{(c+a)-2b}$
$= x^{\circ} \times z^{\circ}$
= 1
So, the value of $x^{b-c} \times y^{c-a} \times z^{a-b}$ is 1

Question 6.
An example of geometric series is
(a) 9, 20, 21, 28
(b) 1, 2, 4, 8
(c) 1, 2, 3, 4
(d) 3, 5, 7, 9

Answer: (b) 1, 2, 4, 8
1, 2, 4, 8 is the example of geometric series
Here common ratio = 2/1 = 4/2 = 8/4 = 2

Question 7.
Three numbers from an increasing GP of the middle number is doubled, then the new numbers are in AP. The common ratio of the GP is
(a) 2
(b) $\sqrt{3}$
(c) $2 + \sqrt{3}$
(d) $2 - \sqrt{3}$

Answer: (c) $2 + \sqrt{3}$
Given that three numbers from an increasing GP
Let the 3 number are: a, ar, ar^2 $(r > 1)$
Now, according to question,
$a, 2ar, ar^2$ are in AP
So, $2ar - a = ar^2 - 2ar$
$\Rightarrow a(2r - 1) = a(r^2 - 2r)$
$\Rightarrow 2r - 1 = r^2 - 2r$
$\Rightarrow r^2 - 2r - 2r + 1 = 0$
$\Rightarrow r^2 - 4r + 1 = 0$
$\Rightarrow r = [4 \pm \sqrt{\{16 - 4 \times 1 \times 1\}}]/2$
$\Rightarrow r = [4 \pm \sqrt{\{16 - 4\}}]/2$
$\Rightarrow r = \{4 \pm \sqrt{12}\}/2$
$\Rightarrow r = \{4 \pm 2\sqrt{3}\}/2$
$\Rightarrow r = \{2 \pm \sqrt{3}\}$
Since $r > 1$
So, the common ratio of the GP is $(2 + \sqrt{3})$

Question 8.
An arithmetic sequence has its 5th term equal to 22 and its 15th term equal to 62. Then its 100th term is equal to
(a) 410

(b) 408
(c) 402
(d) 404

Answer: (c) 402
Let ais the first term and d is the common difference of the AP
Given,
$a_5 = a + (5 – 1)d = 22$
⇒ a + 4d = 22 ………….1
and a15 = a + (15 – 1)d = 62
⇒ a + 14d = 62 ………2
From equation 2 – 1, we get
62 – 22 = 14d – 4d
⇒ 10d = 40
⇒ d = 4
From equation 1, we get
a + 4 × 4 = 22
⇒ a + 16 = 22
⇒ a = 6
Now,
a100 = 6 + 4(100 – 1)
⇒ a100 = 6 + 4 × 99
⇒ a100 = 6 + 396
⇒ a100 = 402

Question 9.
Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If a < b < c and a + b + c = 3/2, then the value of a is
(a) 1/2√2
(b) 1/2√3
(c) 1/2 – 1/√3
(d) 1/2 – 1/√2

Answer: (d) 1/2 – 1/√2
Given, a, b, c are in AP
⇒ 2b = a + c
⇒ b = (a + c)/2 ………….. 1
Again given, a^2, b^2, c^2 are in GP then $b^4 = a^2 c^2$
⇒ $b^2 = \pm ac$ ………… 2
Using 1 in a + b + c = 3/2, we get
3b = 3/2

⇒ $b = 1/2$
hence $a + c = 1$
and $ac = \pm 1/4$
So a & c are roots of either $x2 - x + 1/4 = 0$ or $x^2 - x - 1/4 = 0$
The first has equal roots of $x = 1/2$ and second gives $x= (1 \pm \sqrt{2})/2$ for a and c
Since $a < c$,
we must have $a = (1-\sqrt{2})/2$
⇒ $a - 1/2 - \sqrt{2}/2$
⇒ $a - 1/2 - \sqrt{2}/(\sqrt{2}\times\sqrt{2})$
⇒ $a - 1/2 - 1/\sqrt{2}$

Question 10.
If the positive numbers a, b, c, d are in A.P., then abc, abd, acd, bcd are
(a) not in A.P. / G.P. / H. P.
(b) in A.P.
(c) in G.P.
(d) in H.P.

Answer: (d) in H.P.
Given, the positive numbers a, b, c, d are in A.P.
⇒ 1/a, 1/b, 1/c, 1/d are in H.P.
⇒ 1/d, 1/c, 1/b, 1/a are in H.P.
Now, Multiply by abcd, we get
abcd/d, abcd/c, abcd/b, abcd/a are in H.P.
⇒ abc, abd, acd, bcd are in H.P.

Question 11.
Let Tr be the rth term of an A.P. whose first term is a and the common difference is d. If for some positive integers m, n, $m \neq n$, $T m = 1/n$ and $T n = 1/m$ then (a-d) equals to
(a) 0
(b) 1
(c) 1/mn
(d) 1/m + 1/n

Answer: (a) 0
Given the first term is a and the common difference is d of the AP
Now, $Tm = 1/n$
⇒ $a + (m - 1)d = 1/n$ ………… 1
and $Tn = 1/m$
⇒ $a + (n - 1)d = 1/m$ ………. 2
From equation 2 – 1, we get

$(m - 1)d - (n - 1)d = 1/n - 1/m$
$\Rightarrow (m - n)d = (m - n)/mn$
$\Rightarrow d = 1/mn$
From equation 1, we get
$a + (m - 1)/mn = 1/n$
$\Rightarrow a = 1/n - (m - 1)/mn$
$\Rightarrow a = \{m - (m - 1)\}/mn$
$\Rightarrow a = \{m - m + 1)\}/mn$
$\Rightarrow a = 1/mn$
Now, $a - d = 1/mn - 1/mn$
$\Rightarrow a - d = 0$

Question 12.
In the sequence obtained by omitting the perfect squares from the sequence of natural numbers, then 2011th term is
(a) 2024
(b) 2036
(c) 2048
(d) 2055

Answer: (d) 2055
Before 2024, there are 44 squares,
So, 1980th term is 2024
Hence, 2011th term is 2055

Question 13.
If the first term minus third term of a G.P. = 768 and the third term minus seventh term of the same G.P. = 240, then the product of first 21 terms =
(a) 1
(b) 2
(c) 3
(d) 4

Answer: (a) 1
Let first term = a
and common ratio = r
Given, $a - ar^2 = 768$
$\Rightarrow a(1 - r^2) = 768$
and $ar^2 - ar^6 = 240$
$\Rightarrow ar^2 (1 - r^4) = 240$

Dividing the above 2 equations, we get

$ar^2 (1 - r^4)/a(1 - r^2) = 240/768$

$\Rightarrow \{ar^2 (1 - r^2) \times (1 + r^2)\}/a(1 - r^2) = 240/768$

$\Rightarrow 1 + r^2 = 0.3125$

$\Rightarrow r^2 = 0.25$

$\Rightarrow r^2 = 25/100$

$\Rightarrow r^2 = \sqrt{(1/4)}$

$\Rightarrow r = \pm 1/2$

Now, $a(1 - r^2) = 768$

$\Rightarrow a(1 - 1/4) = 768$

$\Rightarrow 3a/4 = 768$

$\Rightarrow 3a = 4 \times 768$

$\Rightarrow a = (4 \times 768)/3$

$\Rightarrow a = 4 \times 256$

$\Rightarrow a = 1024$

$\Rightarrow a = 2^{10}$

Now product of first 21 terms = $(a^2 \times r^{20})^{10} \times a \times r^{10}$

$= a^{21} \times r^{210}$

$= (2^{10})^{21} \times (1/2)^{210}$

$= 2^{210} / 2^{210}$

$= 1$

Question 14.

If the sum of the first 2n terms of the A.P. 2, 5, 8, ….., is equal to the sum of the first n terms of the A.P. 57, 59, 61, ….., then n equals

(a) 10

(b) 12

(c) 11

(d) 13

Answer: (c) 11

Given, the sum of the first 2n terms of the A.P. 2, 5, 8, ….. = the sum of the first n terms of the A.P. 57, 59, 61, ….

$\Rightarrow (2n/2) \times \{2 \times 2 + (2n - 1)3\} = (n/2) \times \{2 \times 57 + (n - 1)2\}$

$\Rightarrow n \times \{4 + 6n - 3\} = (n/2) \times \{114 + 2n - 2\}$

$\Rightarrow 6n + 1 = \{2n + 112\}/2$

$\Rightarrow 6n + 1 = n + 56$

$\Rightarrow 6n - n = 56 - 1$

$\Rightarrow 5n = 55$

$\Rightarrow n = 55/5$
$\Rightarrow n = 11$

Question 15.
If a, b, c are in GP then log a^n, log b^n, log c^n are in
(a) AP
(b) GP
(c) Either in AP or in GP
(d) Neither in AP nor in GP

Answer: (a) AP
Given, a, b, c are in GP
$\Rightarrow b^2 = ac$
$\Rightarrow (b^2)^n = (ac)^n$
$\Rightarrow (b2\ )^n = a^n \times c^n$
$\Rightarrow \log (b^2)^n = \log(an \times cn\ )$
$\Rightarrow \log b^{2n} = \log a^n + \log c^n$
$\Rightarrow \log (b^n)^2 = \log a^n + \log c^n$
$\Rightarrow 2 \times \log b^n = \log a^n + \log c^n$
$\Rightarrow$ log a^n, log b^n, log c^n are in AP

Question 16.
If the nth term of an AP is 3n – 4, the 10th term of AP is
(a) 12
(b) 22
(c) 28
(d) 30

Answer: (c) 28
Given, $a_n = 3n - 2$
Put n = 10, we get
$a_{10} = 3 \times 10 - 2$
$\Rightarrow a_{10} = 30 - 2$
$\Rightarrow a_{10} = 28$
So, the 10th term of AP is 28

Question 17.
If the third term of an A.P. is 7 and its 7 th term is 2 more than three times of its third term, then the sum of its first 20 terms is

(a) 228
(b) 74
(c) 740
(d) 1090

Answer: (c) 740
Let a is the first term and d is the common difference of AP
Given the third term of an A.P. is 7 and its 7th term is 2 more than three times of its third term
$\Rightarrow a + 2d = 7$ ………….. 1
and
$3(a + 2d) + 2 = a + 6d$
$\Rightarrow 3 \times 7 + 2 = a + 6d$
$\Rightarrow 21 + 2 = a + 6d$
$\Rightarrow a + 6d = 23$ ………….. 2
From equation 1 – 2, we get
$4d = 16$
$\Rightarrow d = 16/4$
$\Rightarrow d = 4$
From equation 1, we get
$a + 2 \times 4 = 7$
$\Rightarrow a + 8 = 7$
$\Rightarrow a = -1$
Now, the sum of its first 20 terms
$= (20/2) \times \{2 \times (-1) + (20-1) \times 4\}$
$= 10 \times \{-2 + 19 \times 4)\}$
$= 10 \times \{-2 + 76)\}$
$= 10 \times 74$
$= 740$

Question 18.
If a, b, c are in AP then
(a) $b = a + c$
(b) $2b = a + c$
(c) $b^2 = a + c$
(d) $2b^2 = a + c$

Answer: (b) $2b = a + c$
Given, a, b, c are in AP
$\Rightarrow b – a = c – b$
$\Rightarrow b + b = a + c$
$\Rightarrow 2b = a + c$

Question 19.
If $1/(b + c)$, $1/(c + a)$, $1/(a + b)$ are in AP then
(a) a, b, c are in AP
(b) a^2, b^2, c^2 are in AP
(c) 1/1, 1/b, 1/c are in AP
(d) None of these

Answer: (b) a^2, b^2, c^2 are in AP
Given, $1/(b + c)$, $1/(c + a)$, $1/(a + b)$
$\Rightarrow 2/(c + a) = 1/(b + c) + 1/(a + b)$
$\Rightarrow 2b2 = a^2 + c^2$
$\Rightarrow a^2, b^2, c^2$ are in AP

Question 20.
3, 5, 7, 9, …….. is an example of
(a) Geometric Series
(b) Arithmetic Series
(c) Rational Exponent
(d) Logarithm

Answer: (b) Arithmetic Series
3, 5, 7, 9, …….. is an example of Arithmetic Series.
Here common difference = 5 – 3 = 7 – 5 = 9 – 7 = 2